Time-Domain Method of Lines Applied to Planar Guided Wave Structures

S. NAM, HAO LING, MEMBER, IEEE, AND TATSUO ITOH, FELLOW, IEEE

***Abstract* — A new time-domain method for the analysis of wave propagation and scattering in a planar transmission structure is developed in which the concept of the method of lines is used. The analytical process incorporated along one of the three dimensions has been executed for each line independently (one-dimensional process) or for one set of lines (two-dimensional process) depending on whether or not the structure contains metallic strips at the dielectric interface boundary. A simple numerical example is presented as a demonstration of the above two processes of the method, and its validity is shown by comparison with other data.**

I. Introduction

The TIME-DOMAIN analysis of microwave planar transmission structures provides an alternative to the frequency-domain approach and is also useful for studying the behavior of pulsed signals in structures such as high-speed digital circuits. A typical time-domain analysis requires discretization of a three-dimensional space into a three-dimensional mesh. Usually, a large computer storage and a long computation time are required. An additional problem of these methods is the difficulty in handling open boundaries.

The method proposed in this paper originates from the fact that most of the discontinuities appearing in the planar transmission structures are located on the substrate surface and the space below and above this surface is uniform and homogeneous. We wish to solve the problem by discretizing only in a two-dimensional surface on the substrate where the discontinuity is located. This is possible if the wave-scattering information in the direction perpendicular to the substrate surface is available analytically. The proposed method actually incorporates this process. The method is somewhat similar to a frequency-domain analysis called the method of lines [1].

The method entails the discretization of the structure by a number of lines perpendicular to the substrate surface as shown in Fig. 1. At the specified time, a spatial diagonalization transform of the field distribution at each intersection of these lines with the substrate surface is calculated by Maxwell's equations discretized only in the x and z directions, which are parallel to the substrate surface. The field information in the y direction is obtained analytically at each point and time. This information can be found from the inverse Fourier transform of the solution of the frequency-domain Helmholtz equation in the y direction.

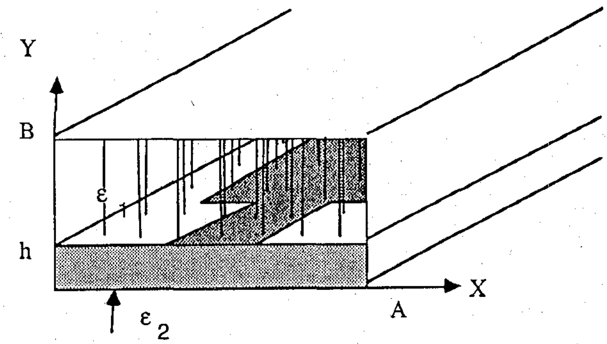

Fig. 1. Typical planar transmission line structure with discontinuity and its discretization example for the analysis.

One may wonder what is happening to the wave-scattering phenomena that are occurring everywhere in the waveguide, not only on each line. This question is natural, because in other time-domain methods the electromagnetic fields at one mesh point interact with those at all six neighboring mesh points in the x, y, and z directions. In the proposed method, the fields at any point on one discretization line do not appear to interact with those on a similar point on another line. It should be emphasized that this is not the case. As we will see shortly, the spatial diagonalization transformation introduced in this method has the property that the real field as a function of (discretized) x and z is transformed to another discretized quantity (transformed field) which contains the real field quantities at all x and z values. Therefore, the analytical information in the y direction in the transformed domain already contains the interaction between lines. Since analytical expressions are used for the field variation in the y direction, this method can easily handle the case where the top wall is removed, whereby the structure is open in the y direction.

II. Method

Let us consider a simple two-dimensional structure as a test case. The formulation for such a structure is simple, yet it contains all essential features of the proposed method. As shown in Fig. 2, the problem is a partially filled rectangular waveguide with/without metallization at the dielectric interface excited by an electric field, E_z, infinite in length and uniform in the z (axial) direction. The

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The authors are with the Department of Electrical and Computer Engineering, University of Texas at Austin, Austin, TX 78712.
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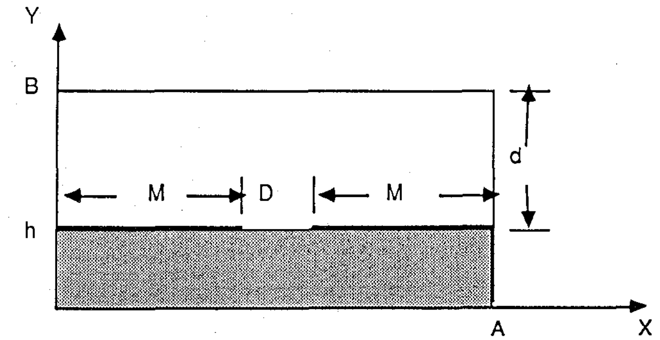

Fig. 2. The cross-sectional structure of a finned rectangular waveguide.

problem is now a two-dimensional one. This problem corresponds to finding the time-domain behavior of the pulsed input in the given structure and the cutoff frequencies of various TM modes in the frequency domain [2].

Because of the excitation, only E_z, H_x, and H_y exist and $\partial/\partial z = 0$. The time-domain equations, discretized in the x *direction only*, are given by

$$-\mu\, \partial[H_x]/\partial t = \partial[E_z]/\partial y \tag{1a}$$

$$-\mu\, \partial[H_y]/\partial t = [D_x^e][E_z]/\Delta x \tag{1b}$$

$$\epsilon(y)\, \partial[E_z]/\partial t = -[D_x^e]^t[H_y]/\Delta x - \partial[H_x]/\partial y \tag{1c}$$

$$[D_{xx}^e][E_z]/(\Delta x)^2 + \partial^2[E_z]/\partial^2 y - \mu\epsilon(y)\, \partial^2[E_z]/\partial^2 t = 0 \tag{1d}$$

where $[D_x^e],[D_{xx}^e]$ are difference operators in which the sidewall boundary condition is incorporated [1] and given by

$$[D_x^e] = \begin{bmatrix} -1 & & 0 \\ & \ddots & \\ 1 & & \\ & \ddots & -1 \\ 0 & & 1 \end{bmatrix}$$

and

$$[D_{xx}^e] = \begin{bmatrix} -2 & 1 & & & 0 \\ -1 & -2 & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & -2 & 1 \\ 0 & & & 1 & -2 \end{bmatrix}. \tag{2}$$

The variables $[E_z]$, $[H_x]$, and $[H_y]$ are the column vectors where the ith element represents the fields along ith line and are functions of y and t.

Since $[D_{xx}^e]$ is a real symmetric matrix, there exists a real orthogonal matrix $[T_x^e]$ that transforms $[D_{xx}^e]$ into a diagonal matrix $[d_{xx}^e]$ and is given by [1]

$$[T_x^e]_{ik} = \sqrt{2/(N+1)}\,\sin[ik\pi/(N+1)] \quad \text{for } i, k = 1 \text{ to } N \tag{3}$$

$$[d_{xx}^e]_{i+1,i} = 2\sin[i\pi/(2N+2)] \quad \text{for } i = 1 \text{ to } N \tag{4}$$

where N is the total number of discretized lines for E_z.

We can now transform $[E_z]$, etc., into a transform $[\widetilde{E_z}] = [T_x^e]^t[E_z]$, etc., where the superscript t stands for transpose. The transform of (1d) is

$$(1/\Delta x)^2[d_{xx}^e][\widetilde{E_z}] + \partial^2[\widetilde{E_z}]/\partial^2 y - \mu\epsilon(y)\, \partial^2[\widetilde{E_z}]/\partial^2 t = 0. \tag{5}$$

Depending on the boundary conditions at the interface, the problems can be classified into two groups.

(I) Problem with no metallization at the interface:

$$[E_z(t, y=h^+)] = [E_z(t, y=h^-)] \quad \text{for all } t \text{ and } i \tag{6a}$$

$$[H_x(t, y=h^+)] = [H_x(t, y=h^-)] \quad \text{for all } t \text{ and } i. \tag{6b}$$

(II) Problem with partially metallized interface:

$$[E_z(t, y=h^+)] = [E_z(t, y=h^-)] \quad \text{for all } t \text{ and } i \tag{7a}$$

$$[H_x(t, y=h^-)] - [H_x(t, y=h^+)] = [J_z(t, y=h)] \quad \text{for all } t \text{ and } i \tag{7b}$$

$$[E_z(t, y=h)] = 0 \quad \text{for all } t \text{ and } i \text{ on } M. \tag{7c}$$

Group (I) contains no metallization at the dielectric interface boundary so that the boundary condition is unchanged along the transformed direction. Group (II) contains metallic strips at the dielectric interface boundary so that the nonuniform boundary condition results along the transformed direction.

A. Uniform Boundary Problem: One-Dimensional Process

Notice that without any metallization at the dielectric interface boundary, the structure in Fig. 2 becomes a partially filled rectangular waveguide. Since the boundary condition is independent of i, (5) can be solved for each i. Using the separation of variable technique, one can obtain a typical Sturm–Liouville differential equation for the y-dependent solution. The solution for the ith line is

$$\widetilde{E_{zi}}(y,t) = \begin{cases} \Sigma_n(A_{ni}\cos\omega_{ni}t + B_{ni}\sin\omega_{ni}t) \\ \quad \cdot \sin K_{1ni}(b-y) \qquad \text{(in region I)} \\ \Sigma_n(A_{ni}\cos\omega_{ni}t + B_{ni}\sin\omega_{ni}t) \\ \quad \cdot (\sin K_{1ni}d/\sin K_{2ni}h)\sin K_{2ni}y \\ \qquad \text{(in region II)} \end{cases} \tag{8}$$

where K_{1ni}, K_{2ni}, and ω_{ni} are determined by the characteristic transcendental equation

$$K_{1ni}\cos K_{1ni}d\sin K_{2ni}h + K_{2ni}\sin K_{1ni}d\cos K_{2ni}h = 0 \tag{9}$$

$$\left[(K_{1ni})^2 - d_{xxi}^3/(\Delta x)^2\right]/\mu\epsilon_1 = \left[(K_{2ni})^2 - d_{xxi}^e/(\Delta x)^2\right]/\mu\epsilon_2 \tag{10}$$

$$\omega_{ni}^2 = \left[(K_{1ni})^2 - d_{xxi}^e/(\Delta x)^2\right]/\mu\epsilon_1. \tag{11}$$

From this point on, there are basically two approaches. If the initial conditions for E_z and its time derivative are given, their diagonalization transforms are readily formed. From those quantities, one can find A_{ni} and B_{ni}. Then, the field at any point at any time can be extracted from the inverse transform of (8) via $[E_z(y,t)] = [T_x^e][\widetilde{E_z}(y,t)]$.

An alternative method is an application of the time-stepping procedure. From the initial condition for E_z and its transform $\widetilde{E_z}$, one can find A_{ni} at time $t=0$ in (8), which will be called A_{ni}^N with $N=0$. With the causality condition, the transforms of (1a)–(1c) can be discretized in time as a time-stepping iteration. Expressing the solution (8) in the form

$$\widetilde{E_{zi}}^N(y) = \begin{cases} \Sigma_n A_{ni}^N \sin K_{1ni}(b-y) \\ \qquad \text{(in region I)} \\ \Sigma_n A_{ni}^N (\sin K_{1ni}d/\sin K_{2ni}h) \sin K_{2ni}y \\ \qquad \text{(in region II)} \end{cases} \tag{12}$$

and similar ones for $\widetilde{H_{xi}}$ and $\widetilde{H_{yi}}$ at the time $(N+1/2)$, one can implement a leapfrog-type iteration scheme to calculate these coefficients. The real field at $y=y_0$ at the Nth time step can be obtained by invoking the inverse transformation as described above to $[\widetilde{E_z}]^N$.

B. Nonuniform Boundary Problem: Two-Dimensional Process

Notice that if the structure contains metallic strips at the dielectric interface boundary, (5) becomes a set of uncoupled partial differential equations related by the nonuniform boundary conditions (7b) and (7c). In this case, the procedure above is no longer applicable. Instead, a set of fields defined on the lines should be found that satisfies the nonuniform boundary condition. Such a set is actually a mode of the structure. Therefore, the first step of the analysis is to find these modes of the structure. Any given input in the transformed domain can be expanded in terms of these modes. The solution of (5) for the ith line of the nth mode which satisfies the transformed boundary condition of (7a) can be written as

$$\widetilde{E_{zni}}(t,y) = \begin{cases} (A_{ni}\cos\omega_{ni}t + B_{ni}\sin\omega_{ni}t)\sin K_{1ni}(b-y) \\ \qquad \text{(in region I)} \\ (A_{ni}\cos\omega_{ni}t + B_{ni}\sin\omega_{ni}t) \\ \quad \cdot(\sin K_{1ni}d/\sin K_{2ni}h)\sin K_{2ni}y \\ \qquad \text{(in region II)} \end{cases} \tag{13}$$

where K_{1ni}, K_{2ni}, and ω_{ni} are related by

$$K_{1ni}^2 = \omega_{ni}^2\mu\epsilon_1 + d_{xxi}^e/(\Delta x)^2$$
$$K_{2ni}^2 = \omega_{ni}^2\mu\epsilon_2 + d_{xxi}^e/(\Delta x)^2. \tag{14}$$

Substituting (13) into the transforms of (1a) and (1b), we can find $\widetilde{H_{xni}}$ and $\widetilde{H_{zni}}$. Therefore, the mode current $[\widetilde{J_{zn}}]$ obtained from the discontinuity of $[\widetilde{H_{xn}}]$ at $y=h$ is

$$\begin{aligned}\widetilde{J_{zni}}(t,\ y=h) &= (-1/\mu)f_{ni}(t)\big[(\sin K_{1ni}d/\sin K_{2ni}h) \\ &\quad \cdot K_{2ni}\cos K_{2ni} + K_{1ni}\cos K_{2ni}d\big] \\ &= \widetilde{Y_{ni}}(t,\ y=h)\widetilde{E_{zni}}(t,\ y=h)\end{aligned} \tag{15}$$

where

$$f_{ni}(t) = (A_{1ni}\sin\omega_{ni}t/\omega_{ni} - B_{1ni}\cos\omega_{ni}t/\omega_{ni})$$
$$\widetilde{Y_{ni}}(t,\ y=h) = (-1/\mu)f_{ni}(t) \cdot(K_{1ni}\cot K_{1ni}d + K_{2ni}\cot K_{2ni}).$$

Now, let us apply the final boundary condition (7b) and (7c) on the real field quantity. Because the boundary condition is position dependent, it cannot be directly applied to the transform quantity. Therefore, it is necessary to obtain the inverse transform of $[\widetilde{J_{zn}}]$:

$$[J_{zn}(t,\ y=h)] = [T_x^e]\big[\widetilde{Y_n}(t,\ y=h)\big] \cdot[T_x^e]^t[E_{zn}(t,\ y=h)] \tag{16}$$

where

$$\big[\widetilde{Y_n}(t,\ y=h)\big] = \text{diag}\big[\widetilde{Y_{ni}}(t,\ y=h)\big].$$

Since $J_{zni}=0$ if the ith line is out of the metallization and $E_{zni}=0$ if the ith line is on the metallization, only the portion of (16) with zero $[J_{zn}]$ and nonzero $[E_{zn}]$ provides the characteristic matrix equation:

$$[J_{zn}(t,\ y=h)]_D = [0] = \Big\{[T_x^e]\big[\widetilde{Y_n}(t,\ y=h)\big][T_x^e]^t\Big\}_{\text{red}} \cdot\big[E_{zn}(t,\ y=h)\big]_D \tag{17}$$

where D implies that the quantity subscribed by D is on the nonmetallization portion of the interface and *red* means the reduced portion of (16) obtained by the method described above. In order for the nth eigenmode, $[E_{zn}]$, to satisfy (17) at all times with a nontrivial solution, the time-dependent expression for each line i should have the same functional form except for a constant factor and the determinant of the reduced matrix should be zero. The final solution of $[\widetilde{E_{zn}}]$ is

$$\widetilde{E_{zni}}(t,y) = \begin{cases} (A_n\cos\omega_n t + B_n\sin\omega_n t)C_{ni}\sin K_{1ni}(b-y) \\ \qquad \text{(in region I)} \\ (A_n\cos\omega_n t + B_n\sin\omega_n t) \\ \quad \cdot C_{ni}(\sin K_{1ni}d/\sin K_{2ni}h)\sin K_{2ni}y \\ \qquad \text{(in region II)} \end{cases} \tag{18}$$

where ω_n is the nth eigenvalue of the characteristic equation given by

$$\text{Det}\Big\{[T_x^e][Y_n(t,\ y=h)][T_x^e]^t\Big\}_{\text{red}} = 0 \tag{19}$$

and C_{ni} can be derived from the corresponding eigenvector, $[E_{zn}]$, of (17).

The procedure described above is very similar to the frequency-domain method of lines [1], and the nth eigen-

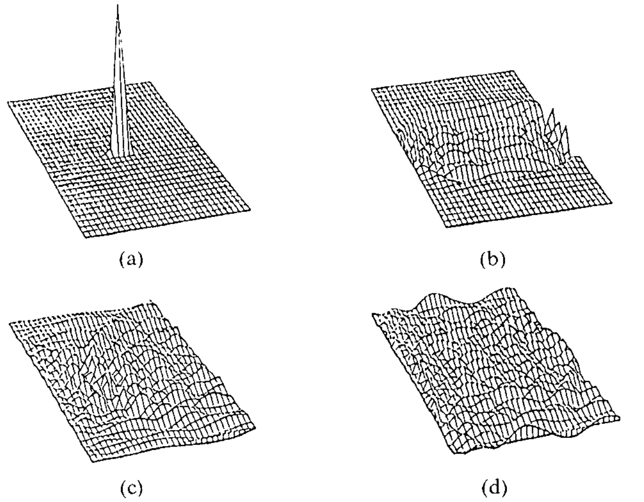

Fig. 3. Distribution of E_z field at various times obtained by one-dimensional process. (a) $t=0$, (b) $t=20$, (c) $t=40$, and (d) $t=60$ [ps].

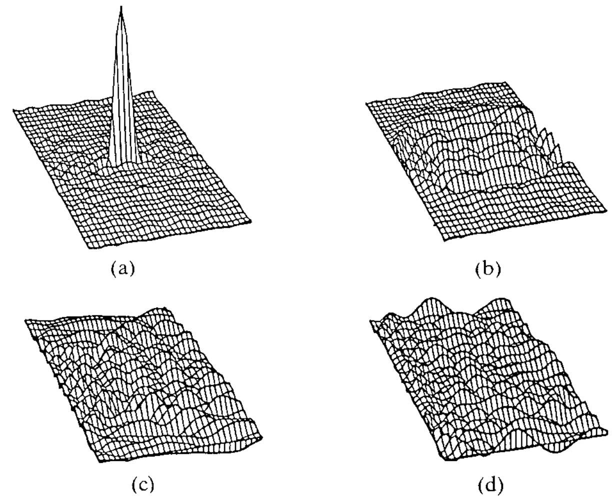

Fig. 4. Distribution of E_z field at various times obtained by two-dimensional process. (a) $t=0$, (b) $t=20$, (c) $t=40$, and (d) $t=60$ [ps].

value, ω_n, is actually the nth cutoff frequency of the given structure. However, the main difference is that we find an eigenvector to obtain a two-dimensional eigenmode of a given structure.

Finally, the transform of any real field can be expanded in terms of the eigenmodes in the transformed domain:

$$\widetilde{E_{zi}}(t, y)=\begin{cases}\Sigma_n(A_n\cos\omega_n t+B_n\sin\omega_n t)C_{ni}\sin K_{1ni}(b-y) \\ \qquad\qquad\text{(in region I)} \\ \Sigma_n(A_n\cos\omega_n t+B_n\sin\omega_n t) \\ \quad\cdot C_{ni}(\sin K_{1ni}d/\sin K_{2ni}h)\sin K_{2ni}y \\ \qquad\qquad\text{(in region II)}.\end{cases} \tag{20}$$

The orthogonality property can be used to find A_n from the initial condition for $[\widetilde{E_z}(t=0, y)]=[T_x^e]^t[Ez(t=0, y)]$. Also, the causality condition assumed by the time iteration method [3] can be used to find B_n:

$$B_n=A_n\tan(\omega_n\Delta t/2)$$

where

$$\Delta t=\Delta x\sqrt{2}\sqrt{\mu\epsilon_{\text{lower}}}. \tag{21}$$

The real field at $y=y_0$ at any time can be obtained by invoking the inverse transformation to the transformed field $[\widetilde{E_z}(t, y=y_0)]$. A similar procedure can be used for the TE excitation problem. Even though this two-dimensional process described above is developed for the analysis of the structure with a nonuniform boundary condition, it can also be applied to the problem with a uniform boundary and thus constitutes a generalization of the one-dimensional process described earlier.

III. Results and Discussion

The accompanying figures are the result of sample calculations. For comparison, the one-dimensional and two-dimensional processes are used to calculate the E_z field distribution in the partially filled rectangular waveguide after a pulsed E_z excitation is imposed at $t=0$ at the center of the cross section. Figs. 3 and 4 show, respectively, the results obtained by the two different processes. Even though the ripples in the two-dimensional process seem to be slightly higher than those of the one-dimensional process, our results show that both processes can be used to analyze the wave propagation characteristics in the time domain. One thing to be noticed here is that although the one-dimensional process is simple and efficient, it cannot be applied to problems with nonuniform boundary conditions. Fig. 5 shows the spectrum of the time signal for E_z obtained by the one-dimensional process, where the waveguide cutoff frequencies correspond to the peaks in the spectrum. The results differ by less than 1 percent from the analytical values. For confirmation of the two-dimensional process, the eigenfrequency of the finned rectangular waveguide for the dominant TE mode is calculated by the characteristic equation (19) and compared with Hoefer's result [4] in Fig. 6. Good agreement is obtained. The result for pulse propagation and scattering in the finned rectangular waveguide obtained by the two-dimensional process is shown in Fig. 7.

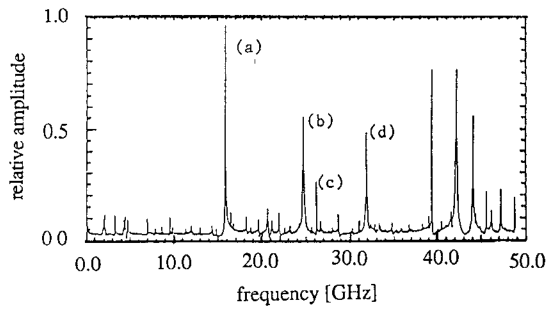

Fig. 5. Cutoff frequency spectrum for the partially filled rectangular waveguide ($A=2$, $B=1$, $h=0.2$ [cm], $\epsilon_1=1$, $\epsilon_2=3$). (a) TM_{11}, (b) TM_{31}, (c) TM_{12} and (d) TM_{32} etc.

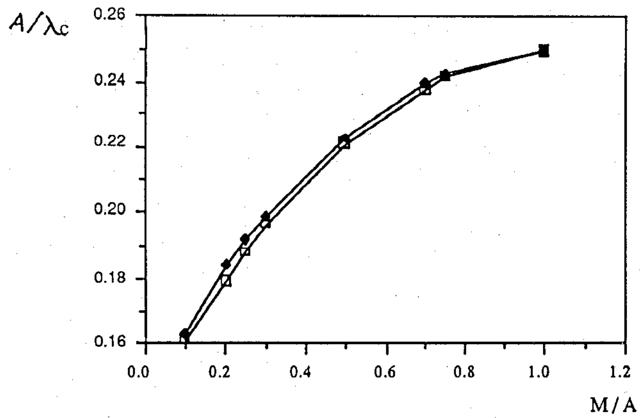

Fig. 6. Cutoff frequency of the finned waveguide shown in Fig. 1 with $B/A = 2$, $h = 1$. (Light squares: present method. Dark squares: Hoefer [4].)

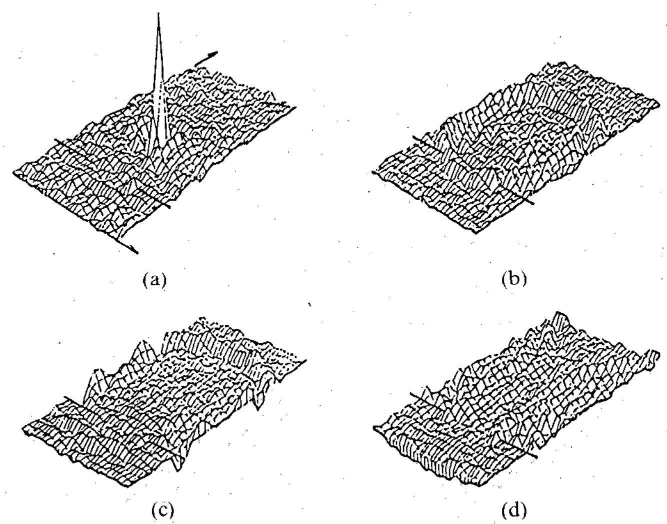

Fig. 7. The picture of a pulse propagation in a finned rectangular waveguide ($A = 1$, $B = 2$, $h = 0.5$ [cm], and $\epsilon_r = 3$) at time (a) $t = 0$, (b) $t = 30$, (c) $t = 40$, and (d) $t = 60$ [ps].

IV. Conclusions

In this paper, we showed that the time-domain method of lines can be used to analyze planar transmission structures. It is accomplished by the one-dimensional or the two-dimensional eigenmode expansion concept depending on the uniformity of the interface boundary condition along the transformed direction. The proposed time-domain method can be applied to analyze three-dimensional wave propagation problems in the time domain.

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S. Nam was born in Korea on February 2, 1959. He received the B.S. degree in electronic engineering from the Seoul National University in 1981 and the M.S. degree in electrical engineering from the Korea Advanced Institute of Science and Technology in 1983. Since 1986, he has been with the Department of Electrical and Computer Engineering, University of Texas at Austin, where he is working towards the Ph.D. degree in microwave engineering. His research interests are in the area of microwave fields and circuits, in particular guided wave structures and their discontinuity effects on microwave circuits.

Hao Ling (S'83–M'86) was born in Taichung, Taiwan, on September 26, 1959. He received the B.S. degrees in electrical engineering and physics from the Masschusetts Institute of Technology in 1982, and the M.S. and Ph.D. degrees in electrical engineering from the University of Illinois at Urbana-Champaign in 1983 and 1986, respectively.

In 1982, he was associated with the IBM Thomas J. Watson Research Center, Yorktown Heights, NY, where he conducted low-temperature experiments in the Josephson Department. While in graduate school at the University of Illinois, he held a research assistantship in the Electromagnetics Laboratory as well as a Schlumberger Fellowship. In 1986, he joined the Department of Electrical and Computer Engineering at the University of Texas at Austin as an Assistant Professor. He participated in the Summer Visiting Faculty Program in 1987 at the Lawrence Livermore National Laboratory. His current research interests include radar cross section analysis of partially open cavities and the characterization of microstrip discontinuities.

Dr. Ling is a recipient of the 1987 National Science Foundation Presidential Young Investigator Award.

Tatsuo Itoh (S'69–M'69–SM'74–F'82) received the Ph.D. degree in electrical engineering from the University of Illinois, Urbana, in 1969.

From September 1966 to April 1976, he was with the Electrical Engineering Department, University of Illinois. From April 1976 to August 1977, he was a Senior Research Engineer in the Radio Physics Laboratory, SRI International, Menlo Park, CA. From August 1977 to June 1978, he was an Associate Professor at the University of Kentucky, Lexington. In July 1978, he joined the faculty at the University of Texas at Austin, where he is now a Professor of Electrical Engineering and Director of the Electrical Engineering Research Laboratory. During the summer of 1979, he was a guest researcher at AEG-Telefunken, Ulm, West Germany. Since September 1983, he has held the Hayden Head Centennial Professorship of Engineering at the University of Texas. In September 1984, he was appointed Associate Chairman for Research and Planning of the Electrical and Computer Engineering Department. He also holds an Honorary Visiting Professorship at the Nanjing Institute of Technology, in China.

Dr. Itoh is a member of Sigma Xi and of the Institute of Electronics and Communication Engineers of Japan. He is a member of Commission B and Chairman of Commission D of USNC/URSI. He served as the Editor of the IEEE Transactions on Microwave Theory and Techniques for 1983–1985. He serves on the Administrative Committee of the IEEE Microwave Theory and Techniques Society. Dr. Itoh is a Professional Engineer registered in the state of Texas.